# NAVAL POSTGRADUATE SCHOOL

**MONTEREY, CALIFORNIA**

# THESIS

**AN ANALYSIS OF COUNTERINSURGENCY CAMPAIGNS USING LANCHESTRIAN BASED MARKETING DIFFERENTIAL EQUATIONS**

by

Christian W. Blasy

September 2010

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| **REPORT DOCUMENTATION PAGE** | *Form Approved OMB No. 0704-0188* |
|---|---|



| **1. AGENCY USE ONLY *(Leave blank)*** | **2. REPORT DATE** September 2010 | **3. REPORT TYPE AND DATES COVERED** Master's Thesis |
|---|---|---|

| | |
|---|---|
| **4. TITLE AND SUBTITLE** An Analysis of Counterinsurgency Campaigns Using Lanchestrian Based Marketing Differential Equations | **5. FUNDING NUMBERS** |
| **6. AUTHOR** **Christian W. Blasy** | |
| **7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)** Naval Postgraduate School Monterey, CA 93943-5000 | **8. PERFORMING ORGANIZATION REPORT NUMBER** |
| **9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)** N/A | **10. SPONSORING/MONITORING AGENCY REPORT NUMBER** |

**11. SUPPLEMENTARY NOTES** The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. IRB Protocol number _______N/A_________.

| **12a. DISTRIBUTION / AVAILABILITY STATEMENT** Approved for public release; distribution is unlimited | **12b. DISTRIBUTION CODE** |
|---|---|

**13. ABSTRACT (maximum 200 words)**

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| | |
|---|---|
| **14. SUBJECT TERMS** Counterinsurgency, Campaign Models, Irregular Warfare | **15. NUMBER OF PAGES** 67 |
| | **16. PRICE CODE** |

| **17. SECURITY CLASSIFICATION OF REPORT** | **18. SECURITY CLASSIFICATION OF THIS PAGE** | **19. SECURITY CLASSIFICATION OF ABSTRACT** | **20. LIMITATION OF ABSTRACT** |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | UU |

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

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# AN ANALYSIS OF COUNTERINSURGENCY CAMPAIGNS USING LANCHESTRIAN BASED MARKETING DIFFERENTIAL EQUATIONS

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Submitted in partial fulfillment of the
requirements for the degree of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

**NAVAL POSTGRADUATE SCHOOL**
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# EXECUTIVE SUMMARY

Counterinsurgency is a concept that the United States has become increasingly familiar with throughout the conflicts in Iraq and Afghanistan. As the level of understanding on the battle field has risen, so have efforts of analysts within the analytic community to describe and model counterinsurgency. The difficulty for the analyst is that the population that is the focus of counterinsurgency is an extraordinarily complex entity to model.

Our approach attempts to analyze the population at the heart of a counterinsurgency using a simple model derived from Lanchester equations and the Vidale-Wolfe marketing model. The output of the model is a change in a population's support over time for one of two competitors, the insurgent or the counterinsurgent.

We identify parametric results showing that a small fledgling insurgency left undisrupted by the government will grow over time to a point of major concern for the government. Utilizing existing data from the Colombian insurgency, we test our model and find that (1) it fits the data well and (2) it is well suited to demonstrate the movement of a population away from supporting the government and toward supporting an insurgency.

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# ACKNOWLEDGMENTS

I would like to thank my advisor, Professor Wayne Hughes for his mentorship, encouragement, and assistance throughout the thesis process. Thank you also to Second Reader Professor Tom Lucas for his involvement and feedback throughout the development of the thesis. A special thanks to Professor Jeff Applegate, Professor Michael Jaye, and Professor Michael Atkinson for helping to clarify the model and solving the differential equations.

I also would like to thank the great people at Johns Hopkins University Applied Physics Laboratory, namely Ted Smyth, John Benedict, and Dean Simmons.

Last, but certainly not least, I would like to thank my wife for all of her love and encouragement while at the Naval Postgraduate School. Without her support, any success in my career would not be possible.

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# I. INTRODUCTION

> IW is a complex, “messy,” and ambiguous social phenomenon that does not lend itself to clean, neat, concise, or precise definition.
>
> —Irregular Warfare Joint Operating Concept (2007)

## A. BACKGROUND

Contemporary military conflict throughout the world is largely characterized by conventional military forces combating nonstate actors. These groups take the form of terrorist organizations, criminal gangs, or insurgent forces. Conflict of this nature is collectively referred to as Irregular Warfare, and it is a struggle that is as old as warfare itself. Despite the abundance of historical examples of Irregular Warfare, traditional military forces have unquestionably struggled throughout operations aimed at combating “irregular” threats. In both Iraq and Afghanistan, the United States has failed to prevent or subsequently quell insurgent growth. In large part, the failure at the strategic level of leadership rests on an inability to recognize the trajectory of an Irregular Warfare insurgency. Developing a descriptive model that affords strategically predictive insights may help to better operate in the environment the United States finds itself in today and in the future.

At the heart of the difficulties senior leadership has with Irregular Warfare is an inability to assess, prior to the start of operations, what impact the conflict may have on the population within an area of interest. A usual source for trusted advice is the analytic community within the military, who is accustomed to providing recommendations based on objective, quantitative analysis. In many warfare areas, analysts are able to investigate and model processes, weapon systems, parameters, and physical systems in order to estimate their possible impact on an enemy. These models and computer simulations generate insight from which decision makers can evaluate courses of action in order to make better decisions, given the information at hand. Irregular Warfare is fundamentally different from these traditional arenas of warfare.

At its core, Irregular Warfare is a struggle between an insurgent group and a "traditional" force for control over a population (FM 3-24, 2006). As the Irregular Warfare Joint Operating Concept (IW JOC) depicts in Figure 1, the focus of effort in an irregular conflict moves away from military forces towards relevant populations that need to be influenced or persuaded to support the government forces.

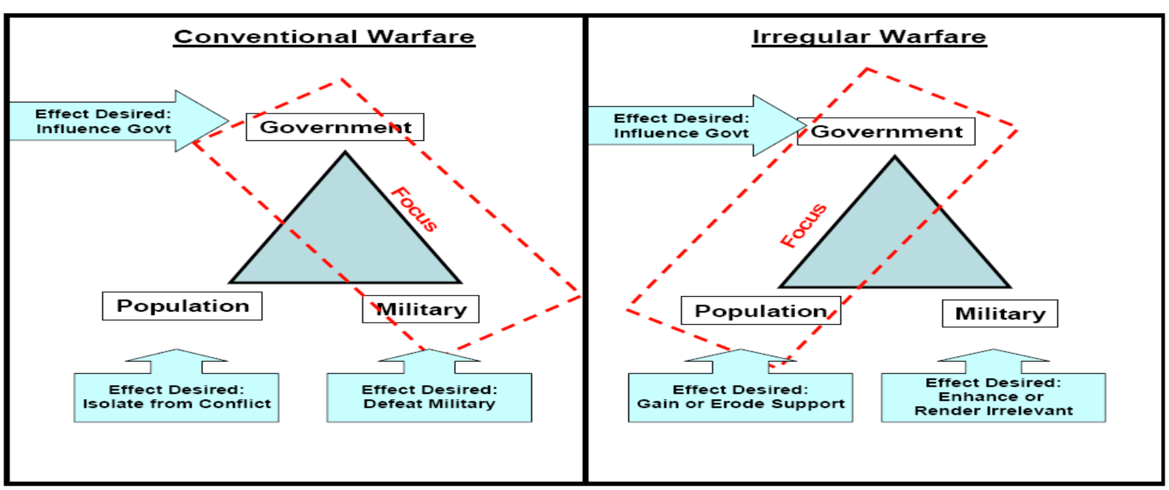


**Figure 1. From IW JOC Contrasting Conventional and Irregular Warfare**

Battles in the irregular environment are not solely focused on killing the enemy and taking land. Soldiers do not assault a beach, tanks do not face tanks, and fighter aircraft are not dueling in the skies. "Irregular" battles are series of interactions, both aggressive and violent, but also peaceful, diplomatic, and humanitarian. From the perspective of the conventional force, every battle must endeavor to influence the vulnerable population toward accepting a benign and stable interaction with a legitimate government. By contrast, the irregular force intends "to isolate their adversaries from the relevant populations and their external supporters, physically as well as psychologically, to bolster their own legitimacy and credibility to exercise authority over that same population" (IW JOC, 2007). Modeling, simulating, and making predictions about these human interactions is hard, and the analytic community within the military has struggled to generate predictive insight. "Irregular warfare is a vast, amorphous concept seething with human psychology and mob behavior. It encompasses politics, economics, psychology, sociology, and most anything else you can think of" (Peck, 2009).

Despite the difficulties Irregular Warfare presents, the analytic community must not lose sight of the fact that Irregular Warfare, like its conventional counterpart, is still a struggle between dueling adversaries. The struggle for legitimacy in the eyes of the population underlying Irregular Warfare can be viewed as each adversary entering a marketplace. Each side offers its product, and begins vying against the competitor for a controlling stake of the market share, the population's support in a counterinsurgency. The goal of this thesis is to introduce a modeling tool, anchored in the rich literature of marketing analysis, where counterinsurgency is described and understood in terms of population influence and control, not attrition.

In Chapter II, we examine some current efforts to model Irregular Warfare and demonstrate the applicability of simple marketing models to gaining insight into a counterinsurgency campaign. Chapter III describes the adapted marketing model and the parameters that define the counterinsurgency campaign. We then solve the differential equation to obtain a steady state value in Chapter IV, as well as examining parametric observations utilizing difference equations of the model. Chapter V makes conclusions from this work and offers some opportunities for future work.

# II. LITERATURE REVIEW AND MODELING BACKGROUND

In Chapter II, we explore some efforts to model the counterinsurgency environment and lay the foundation for our modeling methodology. By understanding the difficulties in current modeling efforts and recognizing the benefits of aggregated models, the value and insight of our simple mathematical model is made evident.

## A. SYSTEM DYNAMIC MODELING

At first glance, the complexities of the Irregular Warfare environment are overwhelming. Interaction between foreign military forces, indigenous forces, civilians, and insurgents is difficult to account for and describe. That difficulty makes modeling the interactions daunting for the analytic community attempting to provide insight to decision makers.

One analysis approach that has been used within the Department of Defense is system dynamic modeling. "System dynamics modeling provides a means of representing the key performance drivers, and their interdependencies and interactions, within dynamically complex businesses and environments." (Mayo & Wichmann, 2003) In 2005, a team from the J8 Warfighting Analysis Division developed a System Dynamic model to represent the elements of Counterinsurgency in FM 3-24, the Field Manual being authored at that time by General David Patraeus and General James Mattis. The team built the layers of the System Dynamic model by stepping through the logical connections between three groups of people: supporters of the host nation government, neutrals, and supporters of the insurgency. The model is depicted in Figure 2.

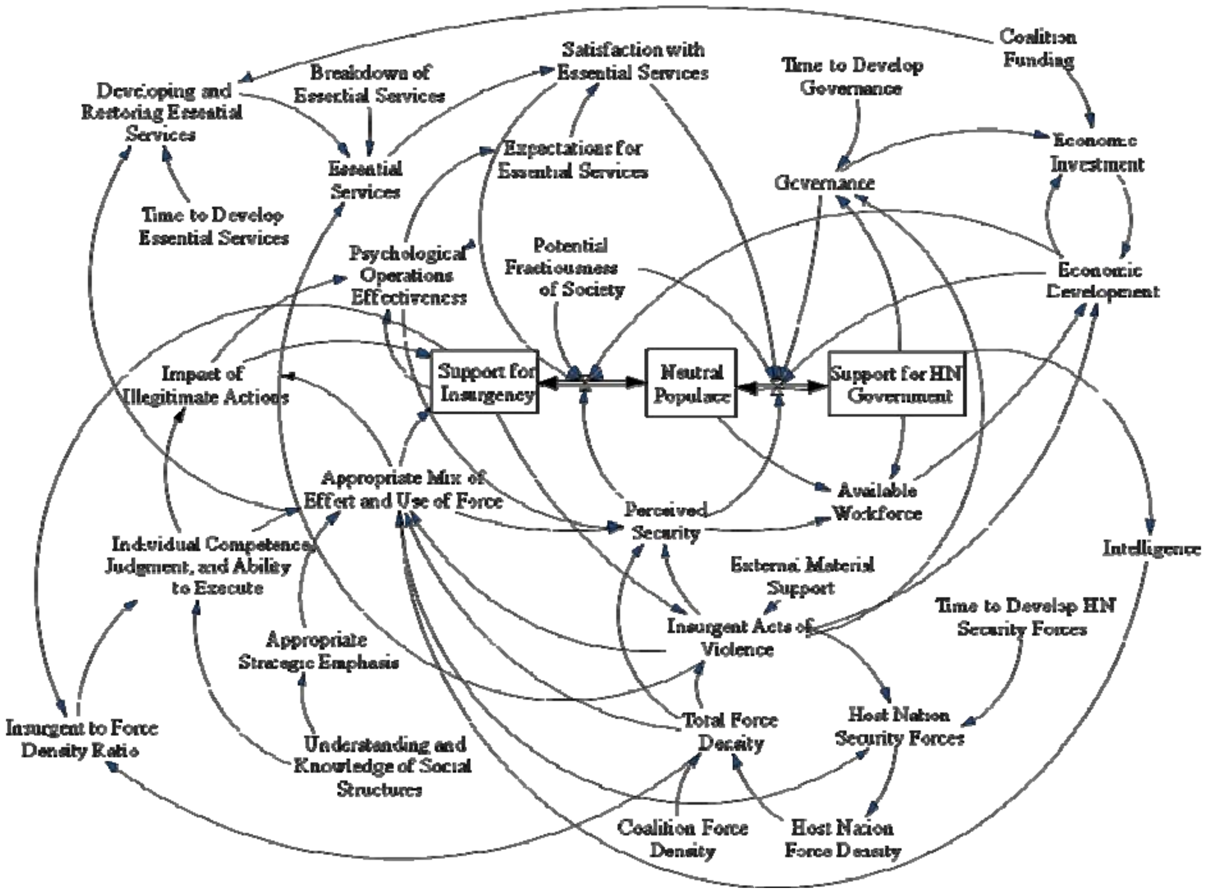


**Figure 2. The Hairball that Stabilized Iraq (From Pierson)**

In a contrasting effort, the United States Marine Corps Combat Development Command, Operations Analysis Division (MCCDC OAD) commissioned a study by Old Dominion University and the Virginia Modeling, Analysis and Simulation Center (VMASC) to provide insight into the Irregular Warfare environment. The study attempted to quantify several aspects of an insurgent environment and apply those values in a system dynamic model, ultimately asserting “that this methodology could be used to analyze contemplated policy changes and their temporal affect on insurgency strength." (Sokolowski et al., 2007)

Like the J8 model, the VMASC model, shown in Figure 3, focused on the impact of events on three populations, when the three were defined as: the Susceptible Population, the Dissident Population, and the Insurgent population.

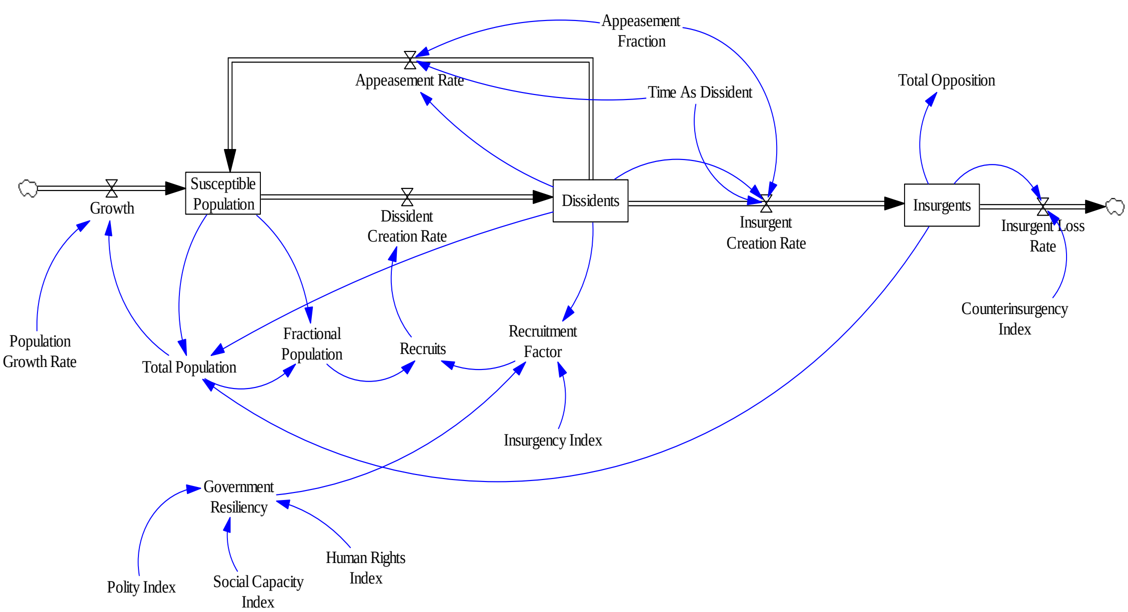


**Figure 3. From VMASC Population Dynamic Model**

In both system dynamic models discussed above, the foundation was a population that was defined by affiliation with the host government or the insurgent forces. Success for either side hinged on the ability to increase its own population by fighting to gain legitimacy within the nation. The outside factors that act upon these populations are the driving forces causing flows between populations. Although these models are competent representations that display logical connections, utilizing them for predictive insight appears dubious. Some of the parameters such as "Recruitment Factor" or "Insurgent Creation Rate" are greatly impacting flows but have little prospect of being populated with reliable numbers. For notional predictive capability, the representation of so many uncertain factors within the undoubtedly complex environment detracts from rather than bolsters credibility. Instead, we model the core of the problem with fewer essential parameters informed by the complex connections but reduced to their simplest dynamic representation of the system. To do so, we start with another simple representation of essential elements, this one of combat dynamics.

## B. LANCHESTER EQUATIONS

### 1. Aimed Fire

Analysis of warfare utilizing mathematical models dates from 1914 with the ground-breaking work of Frederick Lanchester. His development of two sets of differential equations to describe combat has become the foundation on which the majority of ground combat for aggregated models rests. Lanchester modeled what he called "modern" combat, where both forces had the ability to aim and fire at many targets. He contrasted this with ancient warfare, described as sequential duels and later adapted as an "area fire" model (Taylor, 1980).

Lanchester modeled Modern Warfare as two forces, x and y, where both forces would decay at a rate proportional to the individual effectiveness of an enemy soldier. The size of the x force would decay according to the a term below, multiplied by the number of y soldiers. Likewise, the y force size would decay according the individual effectiveness of x soldiers, b, multiplied by the number of x soldiers.

$$\frac{dx}{dt} = -ay$$
$$\frac{dy}{dt} = -bx$$

The relationship is most easily conceptualized in Figure 4, where attrition rate a and force y act on force x, and rate b and force x act on force y.

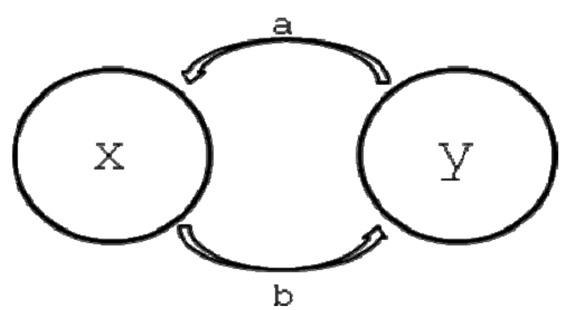


**Figure 4. Lanchester's Model**

### 2. Area Fire

The modern area fire representation is modeled as a function of the number of enemy forces “available” to be killed, multiplied by the forces shooting into the area and their effectiveness per shot fired. In short, the more of both forces involved in the battle, the higher the casualty rate.

$$\frac{dx}{dt} = -axy$$
$$\frac{dy}{dt} = -bxy$$

In both the Aimed Fire (Square Law) and Area Fire (Linear Law) equations, the measure of performance is the rate at which one shooter can kill or wound another. This single value represents the collective combat capability of a force, spread across the number of soldiers engaged in battle. The measure of individual capability is an input to the model that is necessarily derived from the input of many expert considerations, including battlefield data and instrumented experiments not derived from the model itself.

These aggregated assumptions are made in order to reduce something as complex as battles between two opposing forces to some values and relationships that can be understood. It was with such simplifying assumptions, that Lancester demonstrated the advantage of numbers quantitatively, and the benefits of force concentration (Howell, 2007).

> Even though combat between two military forces is a complex random process, such deterministic combat models are commonly used for computational reasons in defense-planning studies. (Taylor, 1980)

## C. IRREGULAR WARFARE MODELS

### 1. Deitchman Model

The Square and Linear Law equations provide analysts and decision makers with valuable insights about conventional force on force engagements, but they offer little toward an understanding of conflict in the Irregular Warfare sense.

In 1962, S.J. Deitchman attempted to extend Lanchester's equations to a situation where guerilla forces would ambush a traditional force. Deitchman modeled the ambushing guerilla force attriting the traditional force with aimed fire while the traditional force was only able to use unaimed area fire to shoot back at the ambushing guerillas (Taylor, 1980). The model was successful in demonstrating that the ambushed force will have a low probability of hitting the concealed ambushing forces, while the guerilla force will have a high probability of hitting the forces being ambushed.

### 2. Kress-Szechtman Model

A recent paper by Moshe Kress and Roberto Szechtman of the Naval Postgraduate School extends the work of Lanchester and Deitchman to model an insurgency (Kress and Szechtman, 2009). Their work models a conflict between a government force and an insurgent force where the intelligence information available to the government drives the attrition suffered by the insurgent force, given by the following equation.

$$\frac{dI}{dt} = -\gamma G(\mu + (1-\mu)\frac{I}{P})$$

where:

$\gamma =$ Attrition coefficient

$G =$ Size of Government

$I =$ Size of Insurgency

$P =$ Size of General Population

$\mu \in [0,1]$ is the level of intelligence

If the intelligence is perfect, μ = 1, the insurgency will suffer attrition according to Lanchester's aimed fire model. At the other extreme, (μ = 0) the insurgency will suffer attrition that is identical to Lanchester's linear law. The intelligence parameter also indirectly affects the ability of the insurgency to attract new recruits. Recruitment is modeled as resulting from government attacks that mistakenly hit the populous, not the insurgents. The likelihood is that attack is affected by inadequate intelligence, μ. The full model is depicted in Figure 5. Insurgents (I) attrite the government (G) according to an aimed fire Lanchester Model. The government is being reinforced (β) while combating the insurgency with the intelligence driven model described earlier. The government campaign against the insurgency causes collateral damage to the population, resulting in additional recruits for the insurgency. The model has the disadvantage that its measure of effectiveness is casualties, with insurgent recruits being solely the result of casualties.

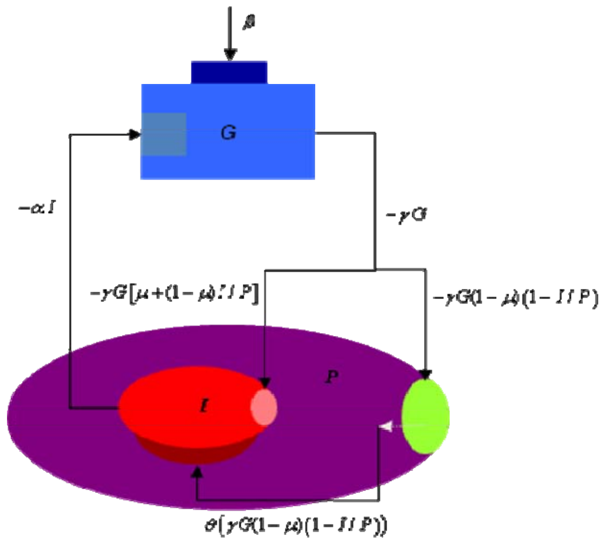


**Figure 5. Kress-Szechtman Model**

## D. MARKETING MODELS

Understanding Lanchester’s equations and extensions of that work along with appreciating the assumptions, modeling considerations and insight they provide, has inspired the development of models with analogous structure for business purposes. Such models serve as the foundation of a model of marketing analysis. In marketing, firms are competing for market share, not killing each other, yet this “flexible class of competitive marketing models. . . have a strong resemblance to Lanchester’s models of warfare.” (Little, 1979)

Mathematical modeling of marketing and advertising strategies and relationships is a rich area of research in the Operations Research community. Analysts in the field of study have “developed mathematical models for many purposes, including better forecasting, integration of data, and understanding markets.” (Shugan, 2002) It is these objectives in marketing research that inspire parallels with Irregular Warfare and counterinsurgency. Understanding the “marketplace” within which forces operate and making predictions about that market is vital to improving the means of mission accomplishment. In business, it is increasing market share. In counterinsurgency, it is inducing the local population to change sides.

Our first example of marketing models is the Vidale–Wolfe model, given by:

$$\frac{dx(t)}{dt} = \rho u(t)(1 - x(t)) - \delta x(t)$$

where *x(t)* is the fraction of the total market share at time *t*, *u(t)* is the amount of advertising expenditure or effort, ρ is the response constant measuring the rate of effectiveness per unit of effort, and δ is the rate at which the market share decays over time due to a diminished interest in, and response to, advertising. (Prasad and Sethi, 2004) “Vidale and Wolfe argued that changes in the rate of sales of a product depend on two effects: response to the advertising that acts on the unsold portion of the market and loss due to forgetting that acts on the sold portion of the market.” (Prasad & Sethi, 2004)

Extending Vidale-Wolfe model to the more relevant and realistic case where competition exists and two competing firms are both vying for market share, we have:

$$\frac{dx_1(t)}{dt} = \rho_1 u_1(t)(1 - x_1(t)) - \rho_2 u_2(t) x_1(t)$$
$$\frac{dx_2(t)}{dt} = \rho_2 u_2(t)(1 - x_2(t)) - \rho_1 u_1(t) x_2(t)$$

Now for each firm *i*=1,2, $x_i(t)$ is the fraction of the total market at time *t*, $u_i(t)$ is the amount of advertising expenditure at some level of effectiveness in winning new customers, and $\rho_i$ a response constant. The major change in the model from the original Vidale-Wolfe model is that the rate at which the market share decays over time is represented by the competitor's efforts to make inroads into his adversary's market share. We no longer have a decay over time due to loss of advertising effectiveness on one's own market, but the possibility of simultaneous stealing of market shares. With the above equations, the model portrays that each competitor will concentrate efforts to increase market share it does not control at time *t* (Bass et al., 2005). The effort to defend the market share already controlled by each competitor is implicit in the coefficients, $u_1$ and $u_2$.

## E. IRREGULAR WARFARE ADAPTATION

The purpose of this thesis is to show that simple mathematical models for marketing analysis can be adapted to study the complex dynamics of irregular warfare. It is true that the Iraqi insurgency or the NATO battle with the Taliban is much more profound than the American market battles between Coca-Cola and Pepsi, but business competition is not simple either. The models that can provide insight to the latter are highly adaptable to the former. What is required for these models to apply and provide valuable insight is principally a change in lexicon. We now examine the terms of the Vidale-Wolfe model of competing firms and describe them for use in Irregular Warfare.

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# III. THE MODEL

> Mathematics, as the language of science, allows interplay between empirical and theoretical research
>
> Steven Shugan

In Chapter III, we define our model and explain in detail the parameters. We utilize counterinsurgency doctrine from FM 3-24 to explain our concept of the “market” in counterinsurgency and explore the terms that influence it.

## A. A LANCHESTRIAN MARKETING MODEL OF IRREGULAR WARFARE

Our model considers the case of Irregular Warfare utilizing equations of the expanded Vidale-Wolfe model in case of two competitors. For the discussion to follow, the competitors are referred to as the Host Nation and the Insurgency. The identification of these two competitors is made in light of a situation, such as that currently being in experienced in Iraq, Afghanistan, the Philippines, and Colombia where U.S. forces are engaged in Irregular Warfare campaigns in cooperation with a host nation against a non state insurgent group.

$$\frac{dm_1(t)}{dt} = \alpha(t)\beta(t)(1-m_1(t)) - \lambda^2(t)(m_1(t))$$

$$\frac{dm_2(t)}{dt} = \lambda^2(t)(1-m_2(t)) - \alpha(t)\beta(t)(m_2(t))$$

Where:

$m_1(t)+m_2(t)=1$, The total population is comprised of a pro-government population and a pro-insurgent population;

$0 \le \alpha \le 1$, The assessed value of the counterinsurgent effort being waged by the government and any assisting government;

$0 \le \beta \le 1$, The assessed value of the government's ability to provide basic services and conditions of good governance;

Units of $\alpha\beta$ is population transferred per unit time.

$0 \le \lambda \le 1$, The assessed value of the insurgency being waged against the government;

Units of $\lambda^2$ ispopulation transferred per unit time.

Utilizing the Lanchestrian model representation depicted in Figure 4, we can represent the Lanchestrian Based Marketing Model with in a similar way.

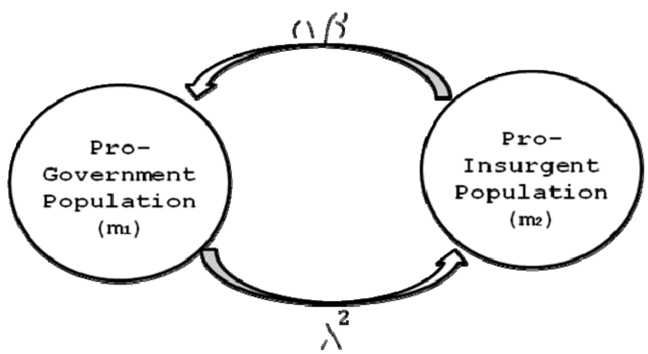


**Figure 6. Lanchestrian Based Marketing Model of IW**

### 1. Market Share *m(t)*

Since Irregular Warfare is a struggle over a relevant population, both the Host Nation and Insurgency control portions of the "market," but one must take care to define the market thoughtfully and clearly in applying our modeling methodology. Control in this case is not a function of the overall "sales," but rather the portion of the relevant population that identifies with, supports, or acquiesces to the given competitor.

A complex and diverse population in an insurgent environment can be defined in many ways, but for clarity here we define a market in one of two ways. First, and most obvious, we can define the market to represent the entire population of the region of concern. This is a relatively large market, taking into account men, women, and children.

The passive majority may or may not have great conviction one way or another regarding the government's battle with the insurgency. The focus of this general population, the grey colored population in Figure 7, is to live in a safe environment, with the essentials of life not in jeopardy. With this sense of the market, the resulting market share reflected in the model corresponds to the popular opinion with regard to the conflict.

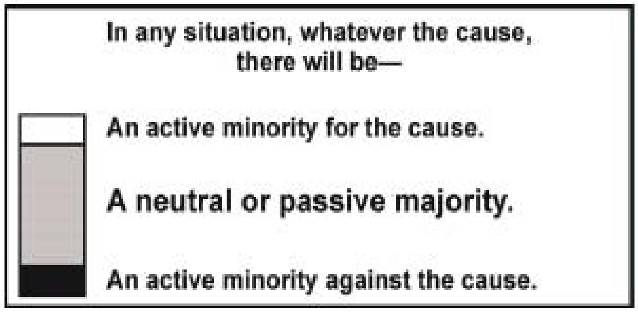


**Figure 7. From FM 3-24 Counterinsurgency Population Depiction**

The second way of defining a market for the purposes of the model is to look at a population in a particular region and seek to represent only the subset of that population that can realistically impact the conflict. In Figure 7, this would be depicted by the white and black populations. This market would represent potential active insurgents or direct supporters. Representing this sense of the market allows the model to produce a sense of the actual end strength of the insurgency. Later we explore both parametric and data driven results utilizing both notions of these population markets.

### 2. Counterinsurgent Effort α

The $\alpha$ term is a number between 0 and 1 that runs in parallel to the $\rho$ term in the Vidale–Wolfe model. The level of advertising that $\rho$ represented in Vidale-Wolfe is, in our model, a parameter that captures the level of counter-insurgent effort. In the advertising world, the $\rho$ value of Coca-Cola would be assessed to be high relative to Shasta Brand cola, as Coca-Cola advertises more than Shasta. The analogous comparison in our model is the large scale counter-insurgency effort in Iraq or Afghanistan vice that currently taking place in the Philippines, where relatively few forces are engaged in action.

### 3. Governmental Effectiveness β

In Vidale-Wolfe, *m(t)* is a value corresponding to the response to marketing for a given firm engaged in advertising. This term is a representation of the attractiveness of a certain product to a population of potential consumers. In the realm of Irregular Warfare, the product being offered is a life controlled either by the host nation or the Insurgents. β is a 0 to 1 bounded value that represents the quality of life for an individual should they choose to support the host nation. β is a measure of effectiveness of the host nation’s ability to provide services, security, and stability. A key point to remember is that this is a value that must represent these factors from the point of view of an individual living in the relevant population, not an outside or independent assessment.

### 4. Insurgency Intensity $\lambda$

The value $\lambda$ represents the intensity, also bounded between 0 and 1, of the insurgency. This intensity can relate to the number or insurgents fighting, the amount of territory or population they control or operate in, or the degree in which they have influence within the established community structure. This effectiveness term is a measure of how persuasive the Insurgents are in their effort to control the population. The sense of this term is less well defined than that of β, but intuitively simple to discern. Lambda can be estimated by the degree with which the Insurgency provides protection or services, such as is the case with a group such as Hezbollah. Lambda can also be derived from the brutality of an Insurgency. Methods such as kidnapping, murder, suicide bombing are all very persuasive when trying to control a relevant population and would correspond to high $\beta$ values. One observation seen through history is that overly brutal tactics can have little or zero effectiveness in controlling a population when they see no way of avoiding that violence.

### 5. Importance of Subject Matter Experts

As was the case with Lanchester's Equations of Modern and Ancient Combat, the quality a of subject matter expert's opinion as an input value in the model is of fundamental importance in our model. This is always the case with insurgency analysis, but there are fewer inputs and therefore less margin for critical error. More complicated models than this one merely compound the need for judgment and possibilities for error. Each of our terms, like the individual attrition terms of Lanchester's Equations, is a composite quantitative representation, a collective sense of a specific situation. Our model serves as a framework for Social Scientists to provide a few critical inputs that capture the essence of each term. Well-founded parameters can then successfully model a given situation, so as to better forecast population dynamics, reassess strategy, or gain insight into enemy actions observed. This rather general description of the importance of subject matter experts is illustrated with abstract examples in later chapters. We show how expert knowledge and opinion from an actual historical irregular war can be applied.

The model is a gross simplification of what is going on during a conflict, of course, but “models that explain all observations often predict poorly. . . stronger approximating assumptions allow cleaner predictions.” (Shugan, 2002,) The model, and its simple framework, is a success, if it provides insight into what is happening with the most crucial entity in Irregular Warfare, the population.

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# IV. MODEL SOLUTION AND ANALYSIS

## A. MODEL SOLUTION

We begin the description of the model solution with our original equation, solving here only for the pro-government (counterinsurgent) population:

$$\begin{aligned}\frac{dm_1(t)}{dt} &= \alpha\beta(1-m_1(t)) - \lambda^2 m_1(t) \\ &= \alpha\beta - (\alpha\beta + \lambda^2)m_1(t)\end{aligned}$$

To solve this equation, we use an integrating factor technique.

$$\frac{dm_1(t)}{dt} + (\alpha\beta + \lambda^2)m_1(t) = \alpha\beta$$

We next multiply both sides by an arbitrary function *f(t).*

$$\frac{dm_1(t)}{dt} f(t) + (\alpha\beta + \lambda^2)m_1(t)f(t) = \alpha\beta f(t)$$

Next, solving for a specific function f(t) such that:

$$(m_1(t)f(t))' = \frac{dm_1(t)}{dt} f(t) + (\alpha\beta + \lambda^2)m_1(t)f(t) = \alpha\beta f(t)$$

This requires solving the easier differential equation.

$$\frac{df}{dt} = (\alpha\beta + \lambda^2) f(t)$$

Whose solution is:

$$f(t) = e^{(\alpha\beta + \lambda^2)t}$$

Allowing us to now solve for:

$$(m_1(t)e^{(\alpha\beta+\lambda^2)t})' = \alpha\beta e^{(\alpha\beta+\lambda^2)t}$$

Integrating both sides yields:

$$m_1(t)e^{(\alpha\beta+\lambda^2)t} = \frac{\alpha\beta}{\alpha\beta+\lambda^2}e^{(\alpha\beta+\lambda^2)t} + C$$

Where *C* is a constant of integration.

Thus: $$m_1(t) = \frac{\alpha\beta}{\alpha\beta+\lambda^2} + Ce^{-(\alpha\beta+\lambda^2)t}$$

Where *C* is set to satisfy the initial condition

$$C = m_1(0) - \frac{\alpha\beta}{\alpha\beta+\lambda^2}$$

The final answer is:

$$m_1(t) = m_1(0)e^{-(\alpha\beta+\lambda^2)t} + (1-e^{-(\alpha\beta+\lambda^2)t})\frac{\alpha\beta}{\alpha\beta+\lambda^2}$$

Therefore, in the steady state the fraction of population that is pro-government (the counterinsurgents) is equal to:

$$\frac{\alpha\beta}{\alpha\beta+\lambda^2}$$

## B. IMPLICATIONS OF THE MODEL

Modeling counterinsurgency within the construct of a competitive marketing environment provides an easily understood and interpreted mental construct for the underlying complexities in an irregular warfare environment.

The parameters within the model represent, and in themselves encompass, a vast amount of observed, researched, or subject expert input. However complicated the input process, the resulting output is straightforward and highly insightful. A junior officer, or the Commanding General, can both understand what the model describes, consistent with existing U.S. Army Doctrine.

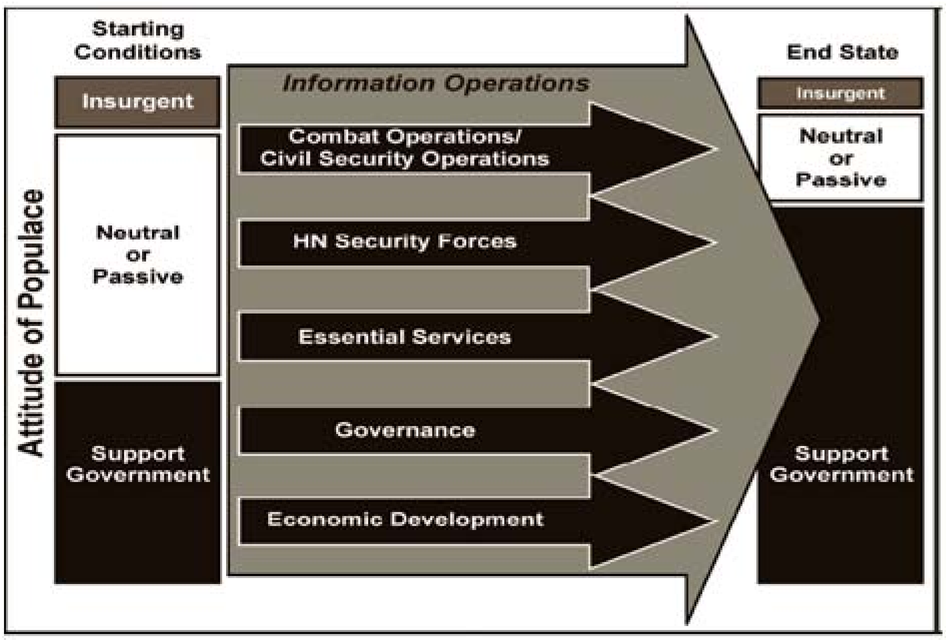


**Figure 8. From FM 3-24 Lines of Operation in IW**

Figure 8 from FM 3-24 depicts the logical lines of operation for a counterinsurgency. These lines are the means with which the relevant population is believed to be influenced into moving from one sub-population to another. Our model closely adheres to these lines of operation, with the parameters $\alpha$ and $\beta$ corresponding directly.

The marketing based model of counterinsurgency also reinforces intuitive beliefs on the environment and tenets espoused in FM 3-24. First, we consider a small group of insurgents, representing a very small portion of a particular population, who are conducting violent acts against local government forces and population centers. Historical knowledge, intuition, and doctrine all assert that despite the small size of the insurgency relative to the population and government forces, the mere existence of such a force will result in increased problems within the region in question.

> A small number of highly motivated insurgents with simple weapons, good operations security, and even limited mobility can undermine security over a large area. Thus, successful COIN operations often require a high ratio of security forces to the protected population. (FM 3-24, pp. 1-2)

### 1. First Model Application

We now apply the model to show how it describes this situation quantitatively. We examine a hypothetical case in which the government of a particular nation is relatively weak in the judgment of experts who assess various factors in a predefined rubric. The relative weakness of the government results in its inability to identify, monitor, apprehend, and prosecute members of an emerging insurgent group. Consistent with the ineffectiveness of security efforts, the government only provides meager support in the way of social services, good governance, and economic vitality to the local populous of 200,000—in which are 3,000 insurgents.

We approximate the differential equation in the full model with a difference equation:

$$\Delta m(t) = \delta(\alpha\beta(1 - m(t)) - \lambda^2(m(t)))$$

where m(t) is the portion of the total population identified as pro-government. We have assigned the model parameters ($\alpha$, $\beta$, and $\lambda$) to reflect the sense of the situation as follows:

$\alpha$ = .4 Level of counter-insurgent effort

$\beta$ = .4 HN services, security, and stability

$\lambda$ = .6 Insurgent level of effort

$\delta$ = .1 Time step

Utilizing these parameters and an initial number of 3,000 insurgents, the resulting growth of the insurgency is seen in Figure 9.

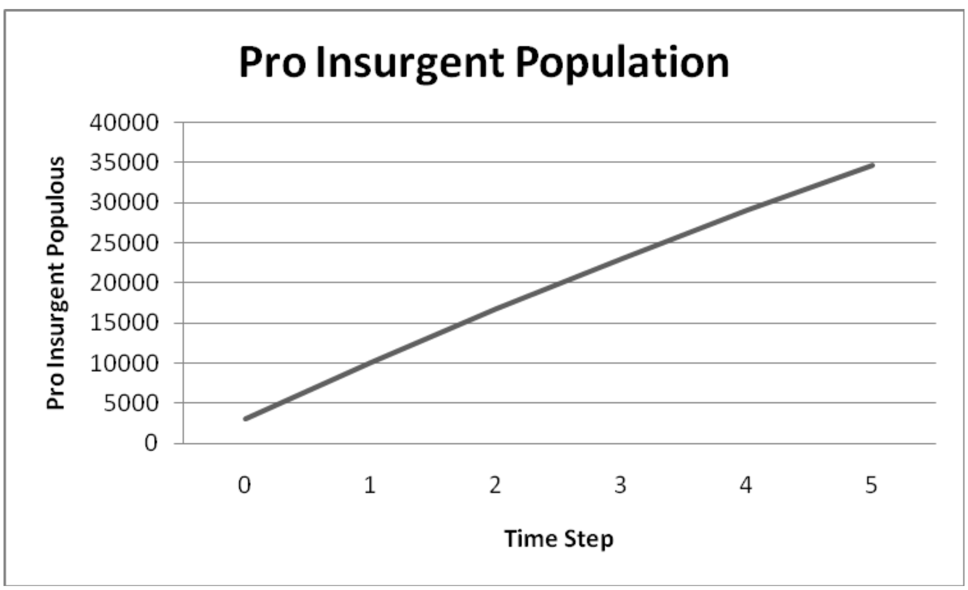


**Figure 9. Insurgent Support over time (Case 1)**

Figure 9 demonstrates that the insurgency quickly will capture a larger portion of the population and grow at a steady rate as the initially small numbers are able to act on and attract members from the larger vulnerable population. On the other hand, the government is battling the insurgency with weak counterinsurgent efforts and offering the population no reason to identify or support its efforts. In a marketing sense, the government is losing market share because of a weak advertising strategy (counterinsurgent effort) and a poor product (social services/economy).

Utilizing the steady-state equation from the complete solution to the differential equation, the long run steady state of the pro-government population, given the conditions, is equal to:

$$\frac{\alpha\beta}{\alpha\beta+\lambda^2}$$

(.4*.4)/(.4*.4 + .6*.6) = .3077

Left unchecked, the pro-insurgent population will grow to almost 70% of the total population, numbering 140,000. The difference equation demonstrates the accuracy of the steady state equation as shown in Figure 10.

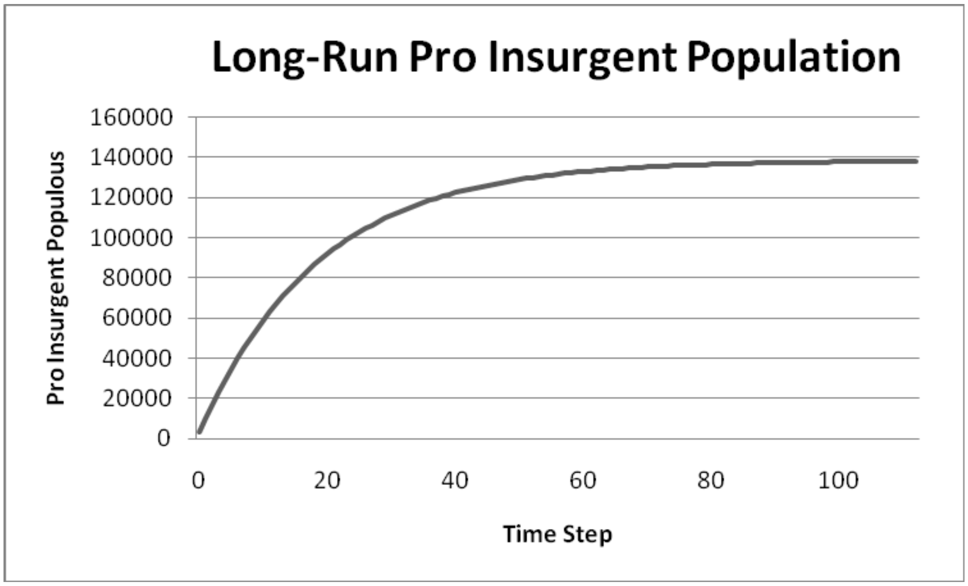


**Figure 10. Insurgent Support over time (Long-Run)**

As the victim nation and possibly outside supporters recognize the growth in the threatening insurgency, efforts are made along the logical lines of operation depicted in Figure 7. The government's level of effort, available resources, and counterinsurgency campaign will not remain constant over time. Active and violent engagement of insurgent forces will take place, resources aimed at nation building and increasing military capacity will be poured into the area, and an increase in the level of host nation troops or outside support troops occurs. The values of the parameters in the model change to reflect the new policy and strategy. The engagement of insurgent fighters will increase the counterinsurgent effort $\alpha$ while also having the simultaneous benefit of decreasing the insurgency effort $\lambda$. The increased resources committed to social service and the economic outlook for the population will have a positive impact on the $\beta$ parameter. The resulting changes in the vignette are listed below.

$\alpha$ = .6 Level of counter-insurgent effort

$\beta$ = .6 HN services, security, and stability

$\lambda$ = .25 Insurgent level of effort

$\delta$ = .1 Time step

Despite the drastic increase in the host nations' parameters reflecting troop increases and resource allocation, and the assault on the ability of the insurgency to have an effective effort, the insurgency still is not beaten or even drastically cut in numbers. The insurgency is barely contained and will persist, as shown in Figure 11, in the painfully slow decline with the given parameters.

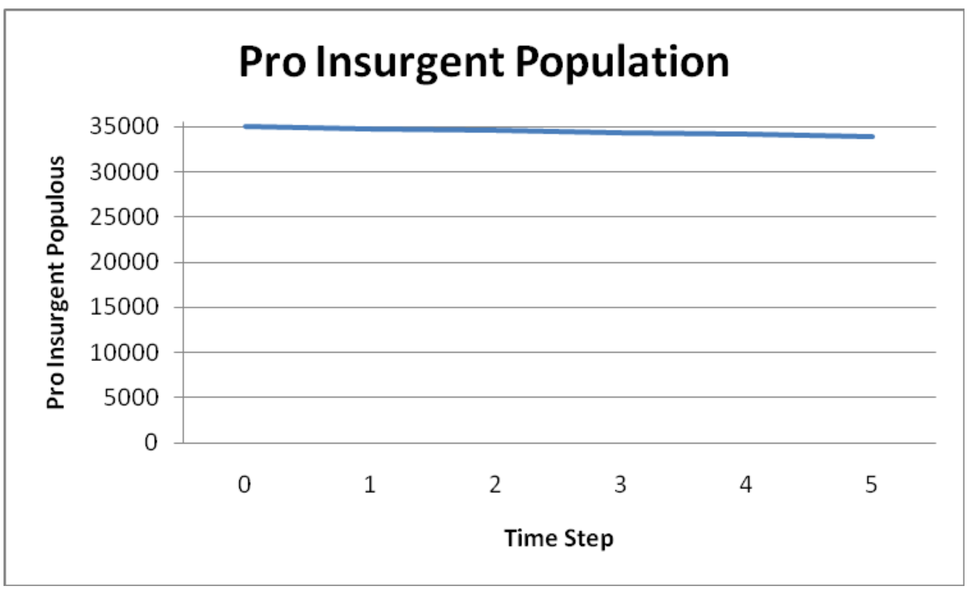


**Figure 11. Insurgent Support over time (Case 2)**

Despite the slow progress made by the government, utilizing the steady state equation from the complete solution to the differential equation, the long run steady state, given the new conditions hold, is equal to:

$$\frac{\alpha\beta}{\alpha\beta+\lambda^2}$$

(.6*.6)/(.6*.6 + .25*.25) = .852

The long-run steady-state equation demonstrates that the pro-government population eventually reaches 85% of the total population. But, it takes a long time to reach equilibrium.

In this finding, demonstrated by a purely hypothetical situation, the model explains the well known motivation behind insurgencies throughout history. Close examination shows it to be a general property of the equations, as well as insurgencies in general. The timeframe for defeating the insurgency is long, even if the parameters in the above scenario are sustainable for the government engaged in battling the insurgency. Nowhere is this more evident than in the current situation the Afghani government and the United States find themselves in fighting the Taliban. Despite an enormous effort by the United States, the relatively small insurgency persists in influencing large portions of the populous with no end in sight.

### C. REAL WORLD APPLICATION OF THE MODEL

Application of the Lanchestrian Based Marketing Model of Irregular Warfare to a hypothetical situation with notional values for the parameters yields interesting and insightful results. However, for the model to have greater credibility, real-world data needs to be applied and analyzed in the context of the equations. As with any model that depends on parameters that reflect an aggregation of subject matter expert's opinions, there is great difficulty in data collection. Fortunately, we are able to employ a study commissioned by USMCCDC OAD to explore our Model of Irregular Warfare with real-world data.

In December 2007, the Virginia Modeling, Analysis, and Simulation Center (VMASC) conducted a study entitled, "Continued Population Dynamics Investigative Research to Support Marine Corps Studies System Irregular Warfare Study." The purpose of the study was to develop a set of indices that could be used as input into the system dynamic model seen in Figure 3, which could then be used to assess the impact of policy changes. The study focuses on the nation of Colombia, and its long struggles with insurgency. In order to study Colombia in the context of the VMASC system dynamic model, the authors defined five indices and a scoring system that would provide numerical inputs into their model. The indices they define are the Polity Index, the Human Rights Index, the Social Capacity Index, the Counterinsurgency Index, and the Insurgency Index. Each of the indices is assessed by subject matter experts who score supporting factors on a scale of 1 to 5 and detracting factors on a scale of -1 to -5.

For the VMSAC study, the experts scored Colombia's indices over two time periods, 1993-2001 and 2001-2006. Table 1 is an example of the index rubric VMASC developed.

| Supporting Factors | 1993-2001 | 2001-2006 |
|---|---|---|
| Military: 40k troops, 55k soldiers, combined mil 125k with proposed increase to 225k | 4 | 5 |
| COIN coupled with War on Terrorism | 4 | 5 |
| Rules of engagement unrestricted | 4 | 5 |
| Citizens support War on Terror approach | 4 | 5 |
| External funding | 4 | 5 |
| Military—not fully capable, but made a clear come-back | 3 | 4 |
| National Police has evolving role | 3 | 4 |
| Uribe taxed the rich and established COIN initiatives | 3 | 5 |
| Legitimacy of counter-insurgency (civilian perspective mixed), fighting anti-democratic forces | 2.5 | 2.5 |
| Military somewhat independent of C-i-C and can be unpredictable | 2 | 1 |
| Civilian army, para-institutionality | 2 | 2 |
| National Police corruption level low | 2 | 3 |
| Stasis | 2 | 2 |
| Spoilers | 2 | 2 |
| Societal based add semblance of democracy | 1 | 2 |
| Supporting Factors Totals | 42.5 | 52.5 |
| Detractions | | |
| Anyone having contacts with guerrillas are legitimate military targets | -5 | -5 |
| Military has conflictual relationship with rural communities | -4 | -4 |
| Counterinsurgency seen as privatized | -3 | -1 |
| Military corruption level high | -3 | -3 |
| Military human rights abuses | -3 | -2 |
| No real defensive strategy | -3 | -2 |
| Spillover of violence into neighboring countries | -3 | -3 |

| | | |
|---|---|---|
| Para militaries treated like felons, tapping resources of National Police | -2 | -2 |
| Military unclear of intent of insurgents | -2 | -2 |
| War fatigue | -2 | -1 |
| Low regard for public / private institutions reinforces vulnerability | -2 | -2 |
| Detraction Total | -32 | -27 |
| Factor Total | 10.5 | 24.5 |

**Table 1. Counterinsurgency Index Scoring Table (From VMASC)**

We apply the input of these experts by first mapping the indices into the parameters of our Lanchestrian Based Marketing Model of Irregular Warfare.

$\alpha$ = Counterinsurgency Index

$\beta$ = Polity Index, Human Rights Index, Social Capacity Index

$\lambda$ = Insurgency Index

We then normalize the indices on a 0 to 1 scale so indices are able to be utilized within our Model of Irregular Warfare.

For the period of 1994–2000 assuming a vulnerable population of just under 85,000 in the Lanchestrian Based Marketing Model of Irregular Warfare, we again approximate the differential equation in the full model with a difference equation whose parameters for Columbia’s case study are:

$\alpha$ = .4856 Level of counter-insurgent effort

$\beta$ = .4284 HN services, security, and stability

$\lambda$ = .5855 Insurgent level of effort

$\delta$ = 1/7 = .1429, (1/years assessed) time step

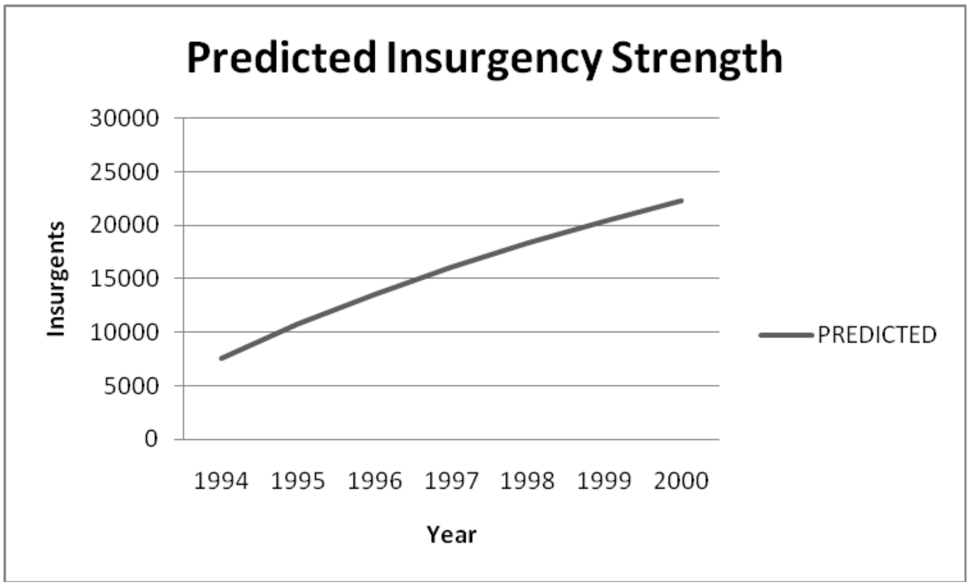


**Figure 12. Predicted Colombian Insurgency (1994-2000)**

Figure 13 demonstrates that the prediction produced by the Lanchestrian Based Marketing Model of Irregular Warfare appears to fit the insurgency strength trend when compared to the actual data on Colombian insurgency strength for the same time period utilized within the VMASC study that had been collected by the Center for Army Analysis.

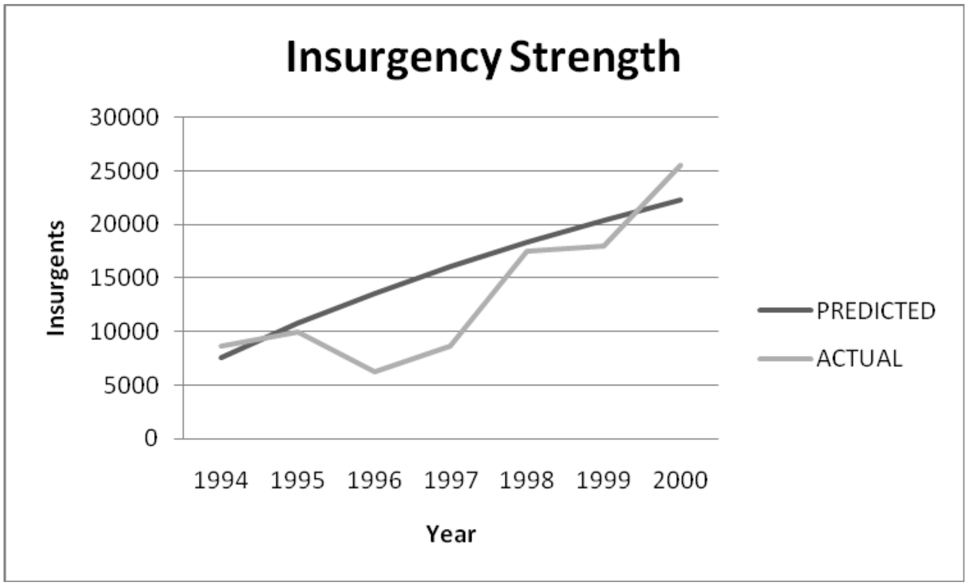


**Figure 13. Predicted Versus Actual Insurgency Strength**

Similar to the situation in the hypothetical vignette previously discussed, the level of effort, available resources and counterinsurgency efforts do not remain constant over time. Policy and strategy of the government change to better address the observed growing threat. For Colombia, that threat was 34,000 insurgents in 2001.

> The U.S. attacks of September 11 paved the way for Colombia’s President Uribe to introduce his state-based, hard line approach to insurgency. U.S. Attorney General John Ashcroft supported the position that drug trafficking and terrorism are the same. This now meant that the counter insurgency once accepted as the war on drugs would be a part of the war against terrorism. (Sokolowski et al., 2007)

Reflecting on the change in policy and strategy, subject matter experts evaluated the changes of each of the indices for the years of 2001-2006. We again apply our Model of Irregular Warfare and examine the resulting trajectory of the counterinsurgency campaign in Figure 14. The index changes are reflected with the parameters for the model as follows:

$\alpha$ = .6202 Level of counter-insurgent effort

$\beta$ = .5359 HN services, security, and stability

$\lambda$ = .4807 Insurgent level of effort

$\delta$ = 1/6 = .1667, (1/years assessed) Time step

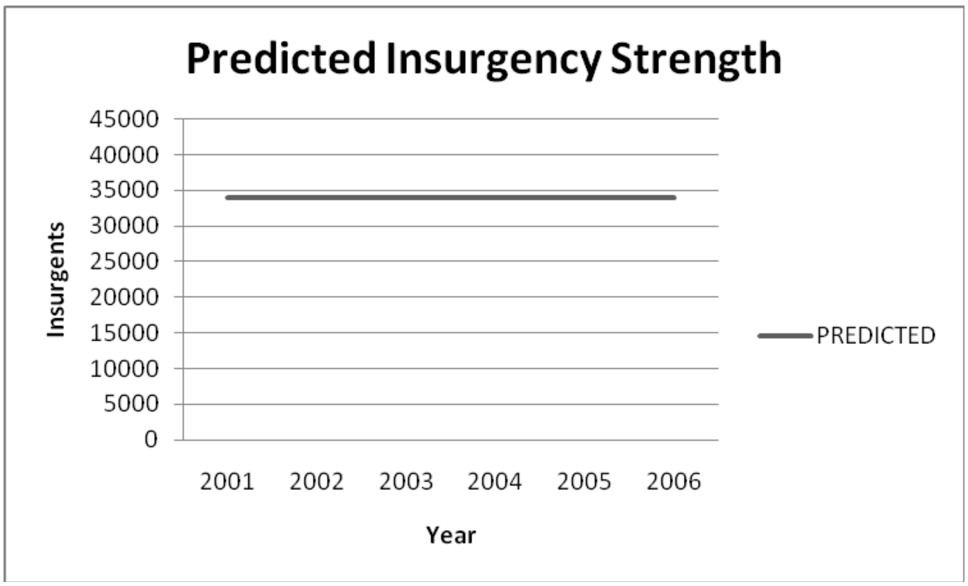


**Figure 14. Predicted Colombian Insurgency**

The prediction produced by the Lanchestrian Based Marketing Model of Irregular Warfare suggests that the battle between the government and the insurgency reaches a virtual stalemate, with the number of insurgents remaining relatively constant over the next six years.

Looking at the anticipated level of insurgency with the actual levels in Figure 15 shows some difference between the values, but the trend over the total time period is not increasing drastically.

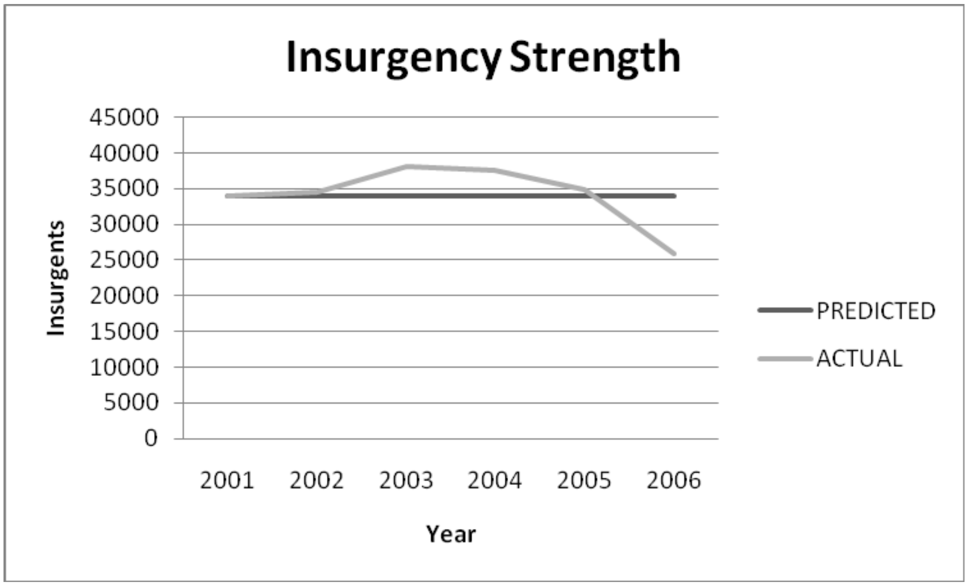


**Figure 15. Colombian Insurgency (2001-2006)**

The large difference between the predicted level of insurgency with the actual level that occurs after 2005 is likely the result of the more than doubling in size of the Colombian military by that time. Since the subject matter experts' scoring of the indices must be applied over the total time period, such a change is not reflected in the model and differences in the out years are expected.

Despite variation in the predicted and actual strength of the Colombian insurgency, when seen over the course of 1994–2006 time period in Figure 16, the Lanchestrian Based Marketing Model of Irregular Warfare offers insight with a notably high level of correspondence to the trend of the insurgency.

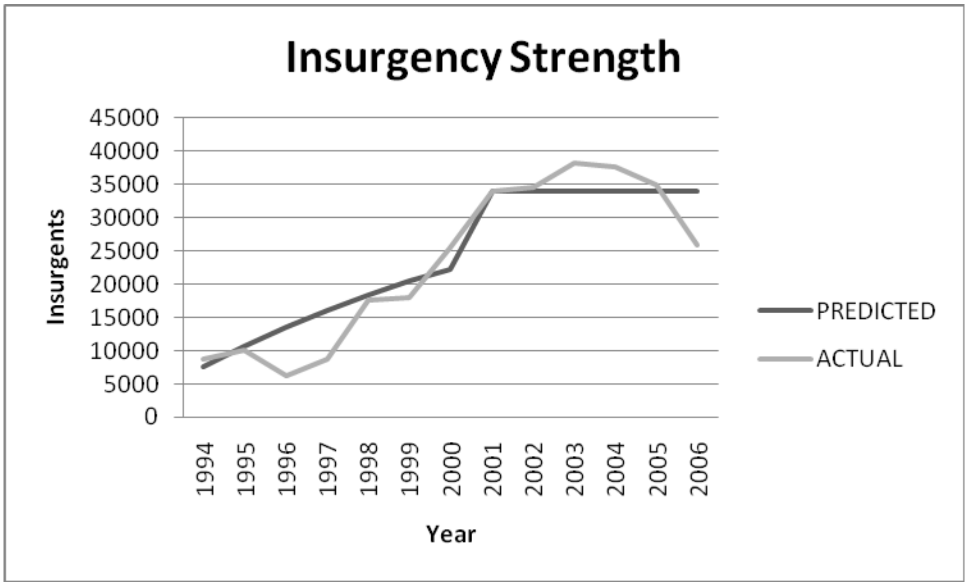


**Figure 16. Colombian Insurgency 1994-2006**

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# V. CONCLUSIONS

## A. RESULTS

This thesis explores the applicability of a Lanchestrian Marketing Model to gain insight into an Irregular Warfare counterinsurgency environment. The insight provided is focused on the popular support for a government battling an insurgency, possibly with the support or presence of U.S. forces, or a rebellious group waging one. The measure of effectiveness in the model is not casualties taken by either side, but rather the trajectory of the population's sentiment or opinion toward the insurgent groups. Gaining insight into that sentiment allows for assessment of how well the counterinsurgent fight is going, because it is from the population where intelligence and ultimately victory are derived.

Rather than attempting to model individual agents or the plethora of cause and effect relationships involved with specific social interactions, our model exploits the simple elegance at the heart of Lanchester's equations. In Chapter II, we explored the class of marketing community models derived from Lanchester's work. In Chapter III, we then describe parallels in the world of Irregular Warfare. We successfully outline parameters that capture the sense of a counterinsurgency environment and provide a framework in which subject matter experts can quantify inputs into a model that is easily understood and interpreted. We solved the resulting differential equations, yielding a steady-state equation that provides immediate insight into the course of a population’s support for the government or the insurgency.

In Chapter IV, we demonstrated parametric results that reinforce known characteristics of insurgency struggles. We next utilized data from a USMC sanctioned study to test our model with an existing data set. The results demonstrated that our model accurately reflects the movement of support within a population toward or away from a government or insurgency.

Perhaps most importantly, our model has demonstrated an ability to focus debate on what is otherwise an ambiguous warfare area for analysts to study. By adapting the input of subject matter experts within the framework of our model, with a small set of parameters that represent an enormous amount of information, interaction, and historical perspective, our model enables easy appreciation and recognition of the impact of policy changes.

## B. FUTURE RESEARCH

The simple nature of our model generally makes verification unneeded; however, validation with data from new and varied sources is necessary to further the credibility of our model.

Future research along the lines of this thesis should seek to identify and utilize data from the current counterinsurgency conflict facing the Afghani and NATO governments in Afghanistan. The circumstances that exist in that conflict are well suited to further test the applicability of our model. Another situation that would seem to provide suitable data to test our model would be the effort of the Philippine government to combat insurgents on the island of Mindanao.

Other opportunities for future research in the applicability of Marketing Models to provide insight into a counterinsurgency should attempt to explore a family of models that have more than two competitors vying for market share. Often in real world counterinsurgency, as in business environments, there are more than two sides competing for market share. The circumstance in regions within Iraq following the U.S. invasion would seem a suitable data source to explore and test the suitability of said models for insight into future conflicts.

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